FOUR PHOTON ENTANGLEMENT FROM DOWN CONVERSION

Harald Weinfurter[1] and Marek Żukowski[2]
[1] *Sektion Physik, Ludwig-Maximilians-Universität, D-80797 München, Germany, and*
Max-Planck-Insitut für Quantenoptik, D-85748 Garching, Germany
[2] *Instytut Fizyki Teoretycznej i Astrofizyki Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*
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Double-pair emission from type-II parametric down conversion results in a highly entangled 4-photon state. Due to interference, which is similar to bunching from thermal emission, this state is not simply a product of two pairs. The observation of this state can be achieved by splitting the two emission modes at beam splitters and subsequent detection of a photon in each output. Here we describe the features of this state and give a Bell theorem for a 4-photon test of local realistic hidden variable theories.

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Parametric down conversion has proven to be the best source of entangled photon pairs so far in an ever increasing number of experiments on the foundations of quantum mechanics [1] and in the new field of quantum communication. Experimental realizations of concepts like entanglement based quantum cryptography [2], quantum teleportation [3] and its variations [4] demonstrated the usability of this source. New proposals for quantum communication schemes [5] and, of course, for improved tests of local hidden variable theories initiated the quest for entangled multi-photon states. Interference of photons generated by independent down conversion processes enabled the first demonstration of a three-photon Greenberger-Horne-Zeilinger (GHZ)-argument [6] and, quite recently, even the observation of a four-photon GHZ-state [7].

In this report we show that four photon entanglement can be obtained directly from type-II parametric down conversion. Instead of sophisticated but fragile interferometric set-ups, we utilize bosonic interference in a double-pair emission process. This effect causes strong correlations between measurement results of the 4 photons and renders type-II down conversion a valuable tool for new multi party quantum communication schemes. The analysis of the entanglement inherent in the four photon emission leads us to a new form of inequality distinguishing local hidden variable theories from quantum mechanics, and demonstrates its potentiality for experiments on the foundations of quantum mechanics.

In type-II parametric down conversion [8] multiple emission events during a single pump pulse lead to the following state

$$Z \exp\left(-i\alpha(a_V^* b_H^* + a_H^* b_V^*)\right) |\, 0\rangle\,, \tag{1}$$

where Z is a normalization constant, α is proportional to the pulse amplitude, and where a_V^* is the creation operator of a photon with vertical polarization in mode a, etc. (Fig. 1). We are interested only in 4 photon effects, i.e. the emission of two pairs. Then only the term in (1) proportional to

$$(a_H^* b_V^* + a_V^* b_H^*)^2 \,|\, 0\rangle\,, \tag{2}$$

is relevant. The particle interpretation of this term can be obtained by its expansion

$$(a_H^{*2} b_V^{*2} + a_V^{*2} b_H^{*2} + 2 a_V^* a_H^* b_V^* b_H^*) \,|\, 0\rangle\,, \tag{3}$$

and is given by the following superposition of photon number states

$$|\, 2H_a, 2V_b\rangle + |\, 2V_a, 2H_b\rangle + |\, 1H_a, 1V_a, 1H_b, 1V_b\rangle\,, \tag{4}$$

where e.g. $2H_a$ means 2 H polarized photons in the beam a.

One should stress here that this type of description is valid only for down conversion emissions, which is detected behind filters endowed with a frequency band, which is narrower than that of the pumping fields [9]. If a wide band down-conversion is accepted then such a state is effective only if counts at the detectors are treated as coincidences, when they occur within time windows narrower than the inverse of the bandwidth of the radiation [10]. If such conditions are not met, then the four photon events are essentially emissions of two independent, entangled pairs, with the entanglement existing only within each pair.

Let us pass the four photon state via two polarization independent $50-50$ beam splitters. For simplicity we assume that at the beam splitters a is transformed into $\frac{1}{\sqrt{2}}(a + a')$ and b into $\frac{1}{\sqrt{2}}(b + b')$, with prime denoting the reflected

beam. One can expand the expression (4), and then extract only those terms that lead to 4 photon coincidence behind the two beam splitters, i.e. only those terms for which there is one photon in each of the beams. The resulting component of the full state is given by

$$[4\,(a^*_H a'^*_H b^*_V b'^*_V + a^*_V a'^*_V b^*_H b'^*_H) + 2(a^*_H a'^*_V + a^*_V a'^*_H)(b^*_V b'^*_H + b^*_H b'^*_V)]\,|\,0\rangle\,. \tag{5}$$

The first term represents a 4-photon GHZ state, whereas the second one is a product state of two EPR-Bohm states (the $|\,\Psi^+\rangle$ Bell states in polarizations H and V). This, after the normalization, can be symbolically written as

$$\sqrt{2/3}\,|\,GHZ\rangle_{aa'bb'} + \sqrt{1/3}\,|\,EPR\rangle_{aa'}\,|\,EPR\rangle_{bb'}\,. \tag{6}$$

For additional simplicity of the presentation we also rotate the polarizations in the beams a and a' by $90°$. Thus now our initial state is given by (6) with the GHZ state in its standard form, resulting in the state

$$\begin{array}{c}\sqrt{1/3}\,(\,|\,VVVV\rangle_{aa'bb'} + |\,HHHH\rangle_{aa'bb'}\\ +\frac{1}{2}\,(|\,HVHV\rangle_{aa'bb'} + |\,HVVH\rangle_{aa'bb'} + |\,VHHV\rangle_{aa'bb'} + |\,VHVH\rangle_{aa'bb'}))\,.\end{array} \tag{7}$$

In order to demonstrate the entanglement of this state let us analyze polarization correlation measurements involving all four exit ports of the beam splitters, where the actual observables to be measured are elliptic polarizations with main axis of the polarization ellipse at $45°$. Such observables are of dichotomic nature, i.e. endowed with two valued spectrum $k = +1, -1$, and are defined for each spatial propagation mode $x = a, a', b, b'$ by their eigenstates

$$\sqrt{1/2}\,|\,V\rangle_x + ke^{-i\phi_x}\sqrt{1/2}\,|\,H\rangle_x = |\,k, \phi_x\rangle\,. \tag{8}$$

The probability amplitudes for the results $k, l, m, n = \pm 1$ at the detector stations in the beams a, a', b, b', under local phase settings $\phi_a, \phi_{a'}, \phi_b, \phi_{b'}$, respectively, are given by

$$\frac{1}{4\sqrt{3}}\left[1 + klmn\,e^{i\sum\phi} + \frac{1}{2}(k\,e^{i\phi_a} + l\,e^{\phi_{a'}})(m\,e^{i\phi_b} + n\,e^{i\phi_{b'}})\right]\,, \tag{9}$$

where $\sum\phi$ stands for the sum of all local phases. Therefore the probability to get a particular set of results (k, l, m, n) is given by

$$\begin{array}{c}P(k,l,m,n|\phi_a,\phi_{a'},\phi_b,\phi_{b'})\\ = \frac{1}{16}\left[\frac{2}{3}(1 + klmn\cos\sum\phi)\right.\\ +\frac{1}{3}(1 + kl\cos(\phi_a - \phi_{a'}))(1 + mn\cos(\phi_b - \phi_{b'}))\\ \left.+\frac{1}{3}Re\left((1 + klmn\,e^{i\sum\phi})(k\,e^{i\phi_a} + l\,e^{i\phi_{a'}})(m\,e^{i\phi_b} + n\,e^{\phi_{b'}})\right)\right]\,.\end{array} \tag{10}$$

The last term is written in the form of a real part of a complex function to shorten the expression.

The correlation function is defined as the mean value of the product of the four local results

$$E(\phi_a,\phi_{a'},\phi_b,\phi_{b'}) = \textstyle\sum_{k=\pm1}\sum_{l=\pm1}\sum_{m=\pm1}\sum_{n=\pm1} klmn\,P(k,l,m,n|\phi_a,\phi_{a'},\phi_b,\phi_{b'})\,. \tag{11}$$

Its explicit form for the considered process is given by

$$E(\phi_a,\phi_{a'},\phi_b,\phi_{b'}) = \tfrac{2}{3}\cos\textstyle\sum\phi + \tfrac{1}{3}\cos(\phi_a - \phi_{a'})\cos(\phi_b - \phi_{b'})\,. \tag{12}$$

Only the first two terms of the probabilities (10) contribute to the correlation function, and the function is itself a weighted sum of the GHZ correlation function (the first term) and a product of two EPR-Bell correlation functions. The last term in (10) gives a zero contribution to the correlation function, because sums like $\sum_{klmn} ln$ vanish.

The correlation function (12) for the process has a more complicated form than in the usual cases for GHZ-type states, but the strong correlations for numerous phase settings clearly indicate incompatibility with local realistic theories. When inserted into Mermin-type Bell inequalities [11] for four particle systems, the violation is not too impressive, even for optimal sets of local phases. However, here we present a reasoning, involving Bell inequalities of a new type [12], giving stronger inequalities for distinguishing the validity of the different theories in a four photon experiment.

In a local hidden variable theory a correlation function has to be modeled by a construction of the following form (see e.g. [13], [14])

$$E_{LHV}(\phi_a,\phi_{a'},\phi_b,\phi_{b'}) = \int d\lambda \rho(\lambda) I_a(\phi_a,\lambda) I_{a'}(\phi_{a'},\lambda) I_b(\phi_b,\lambda) I_{b'}(\phi_{b'},\lambda)\ , \tag{13}$$

where λ represents an arbitrary set of values of local hidden variables, $\rho(\lambda)$ their probabilistic distribution, and $I_x(\phi_x,\lambda) = \pm 1$ ($x = a, a', b, b'$) represent the predetermined values of the measurements. Their values depend on the set of hidden variables and on the value of the *local* phase settings.

We start with allowing each observer of beam x ($= a, a', b, b'$) to choose, just like in the standard cases of the Bell and GHZ theorems [13], [14], between two values ϕ_x^1 and ϕ_x^2 of the local phase settings.

The formula for the LHV correlation function for the chosen settings is given by

$$E_{LHV}(\phi_a^p,\phi_{a'}^q,\phi_b^r,\phi_{b'}^s) = \int d\lambda \rho(\lambda) I_a(\phi_a^p,\lambda) I_{a'}(\phi_{a'}^q,\lambda) I_b(\phi_b^r,\lambda) I_{b'}(\phi_{b'}^s,\lambda) \tag{14}$$

with $p, q, r, s = 1, 2$. It is important to stress that one must consider arbitrary LHV correlation functions. The only constraint being their structure given by (14).

One can treat the full set of the LHV predictions as a four index tensor $\hat{E}_{LHV}$, with the indices $p, q, r, s = 1, 2$, built out of the tensorial products of two dimensional real vectors $\mathbf{v}_x^\lambda = (I_x(\phi_x^1,\lambda), I_x(\phi_x^2,\lambda))$, which represent the two possible results of a given observer for the given value of the hidden variable:

$$\hat{E}_{LHV} = \int d\lambda \rho(\lambda) \mathbf{v}_a^\lambda \otimes \mathbf{v}_{a'}^\lambda \otimes \mathbf{v}_b^\lambda \otimes \mathbf{v}_{b'}^\lambda \tag{15}$$

The actual values of the components of the two dimensional vectors $(I_x(\phi_x^1,\lambda), I_x(\phi_x^2,\lambda))$ can be equal to only either $(1,1)$, or $(1,-1)$, or $(-1,-1)$, or finally $(-1,1)$. Let us denote these four possible vectors by $\mathbf{v_x^j}$ with $j = 1, 2, 3, 4$, respectively. Thus, the LHV correlation function (tensor) can be simplified to a discrete sum over hidden probabilities $p_{k,l,m,n}$ of the tensorial products of all possible measurement results.

$$\hat{E}_{LHV} = \textstyle\sum_{k,l,m,n=1,\ldots,4} p_{k,l,m,n}\, \mathbf{v_a^k} \otimes \mathbf{v_{a'}^l} \otimes \mathbf{v_b^m} \otimes \mathbf{v_{b'}^n} \tag{16}$$

A further simplification of the tensor is possible since $(-1,-1) = -(1,1)$ and $(-1,1) = -(1,-1)$, or in other words $\mathbf{v_x^{k+2}} = -\mathbf{v_x^k}$. The tensorial products $\mathbf{v_a^k} \otimes \mathbf{v_{a'}^l} \otimes \mathbf{v_b^m} \otimes \mathbf{v_{b'}^n}$ with $k, l, m, n = 1, 2$ form a complete orthogonal (product) basis in the (real) Hilbert space of tensors $R^2 \otimes R^2 \otimes R^2 \otimes R^2$. One can thus rewrite the expansion (16) so that it becomes an expansion in terms of the aforementioned basis

$$\hat{E}_{LHV} = \textstyle\sum_{k,l,m,n=1,2} c_{k,l,m,n}\, \mathbf{v_a^k} \otimes \mathbf{v_{a'}^l} \otimes \mathbf{v_b^m} \otimes \mathbf{v_{b'}^n}\ . \tag{17}$$

The relation between the coefficients in (17) and the probabilities of (16) is given by

$$\begin{aligned} c_{k,l,m,n} &= p_{k,l,m,n} - p_{k+2,l,m,n} - p_{k,l+2,m,n} - \ldots \\ &+ p_{k+2,l+2,m,n} + \ldots - p_{k+2,l+2,m+2,n} + \ldots + p_{k+2,l+2,m+2,m+2}\ . \end{aligned} \tag{18}$$

The expansion coefficients are of course *unique*, and since $\sum_{k,l,m,n=1,\ldots,4} p_{k,l,m,n} = 1$ they satisfy the following inequality

$$\textstyle\sum_{k,l,m,n=1,2} |c_{k,l,m,n}| \leq 1\ . \tag{19}$$

This inequality is a necessary condition for the local realistic description to hold, and thus gives the handle for evaluating the validity of this class of theories. It should be stressed that one can also show that the inequality (19) is also a sufficient condition (see [12]).

To compare the structure of the possible LHV correlation functions with our quantum one (12), let us, in order to simplify the analysis, choose specific values for the local phase settings. First, the observer of beam a will be allowed the choice between $\phi_a^1 = 0$ and $\phi_a^2 = \pi/2$. The other observers ($y = a', b, b'$) can choose between $\phi_y^1 = -\pi/4$ and $\phi_y^2 = \pi/4$. Next, one can expand the quantum function (12) into a sum of products of sine and cosine functions of *single* phases

$$\begin{aligned} &E(\phi_a,\phi_{a'},\phi_b,\phi_{b'}) \\ &= c_a c_{a'} c_b c_{b'} + s_a s_{a'} s_b s_{b'} - \tfrac{1}{3}(s_a s_{a'} c_b c_{b'} + c_a c_{a'} s_b s_{b'}) \\ &- \tfrac{2}{3}(s_a c_{a'} s_b c_{b'} + c_a s_{a'} c_b s_{b'} + c_a s_{a'} s_b c_{b'} + s_a c_{a'} c_b s_{b'})\ , \end{aligned} \tag{20}$$

where $s_x = \sin\phi_x$ and $c_x = \cos\phi_x$. For each fixed set of four local settings one can calculate the specific value of the quantum correlation function $E(\phi_a^p,\phi_{a'}^q,\phi_b^r,\phi_{b'}^s)$. We notice, that for the specific phase settings given above one has ($y = a', b, b'$)

$$(\cos(\phi_y^1), \cos(\phi_y^2)) = \frac{1}{\sqrt{2}}(1,1),$$

and

$$(\sin(\phi_y^1), \sin(\phi_y^2)) = -\frac{1}{\sqrt{2}}(1, -1),$$

whereas

$$(\cos(\phi_a^1), \cos(\phi_a^2)) = (1, 0) = \frac{1}{2}(1, 1) + \frac{1}{2}(1, -1)$$

and

$$(\sin(\phi_a^1), \sin(\phi_a^2)) = (0, 1) = \frac{1}{2}(1, 1) - \frac{1}{2}(1, -1).$$

Therefore the quantum predictions can be arranged to form a tensor, too, and it is easy to write down its expansion in the product basis (the same basis as in eq. (17))

$$\hat{E} = \textstyle\sum_{k,l,m,n=1,2} q_{k,l,m,n} \mathbf{v_a^k} \otimes \mathbf{v_{a'}^l} \otimes \mathbf{v_b^m} \otimes \mathbf{v_{b'}^n} \ . \tag{21}$$

The actual values of the expansion coefficients $q_{k,l,m,n}$ can be straightforwardly obtained from (20). However, note that for the specific set of angles chosen above one has

$$\textstyle\sum_{k,l,m,n=1,2} |q_{k,l,m,n}| = \frac{8}{3\sqrt{2}} > 1 \ . \tag{22}$$

Keeping in mind that the expansion in terms of basis vectors is unique, the quantum correlation function out of which the tensor $\hat{E}$ is built thus violates the necessary condition for local realism (19).

The quantum correlation function satisfies (19) *only if* it is multiplied by a scaling factor v equal or smaller than $\frac{3\sqrt{2}}{8} \approx 53\%$, in other words, if one replaces it by

$$E'(\phi_a, \phi_{a'}, \phi_b, \phi_{b'}) = vE(\phi_a, \phi_{a'}, \phi_b, \phi_{b'}) \ . \tag{23}$$

In an interferometry experiment this scaling parameter is directly related to the visibility (contrast) of the interference pattern. Visibilities lower than one can be interpreted as arising due to some noise contribution to the state. If one considers mixed states for the system of the type

$$\rho_v = (1 - v)\rho_{noise} + v \,|\,\boldsymbol{\psi}\rangle\,\langle\boldsymbol{\psi}| \ , \tag{24}$$

where $\rho_{noise} = \frac{1}{16}\hat{I}$ represents completely uncorrelated noise contribution, and $|\,\boldsymbol{\psi}\rangle$ stands for our pure state (6), then the aforementioned critical v gives the threshold beyond which no LHV model can resemble the quantum predictions.

In conclusion, parametric type-II down conversion not only produces entangled photon pairs, but also highly entangled four photon states. The observation of these states is experimentally much easier to achieve than for GHZ-type states. Here, the full set of probabilities of possible LHV-predictions is compared with the quantum predictions resulting in a significant distinction of the theories.

The other interesting feature of the considered process is that for a number of specific settings one obtains perfect four photon correlations. E.g., for all local phases equal to zero the correlation function is equal to 1, whereas for $\phi_b = \phi_{b'}$, $\phi_a - \phi_{a'} = \pi$ and $\phi_{a'} + \phi_b = 0$ it is equal to -1. This directly enables one to transfer the standard protocols for entanglement based quantum cryptography [15] to the four photon case making multi party quantum key distribution and quantum communication complexity schemes feasible.

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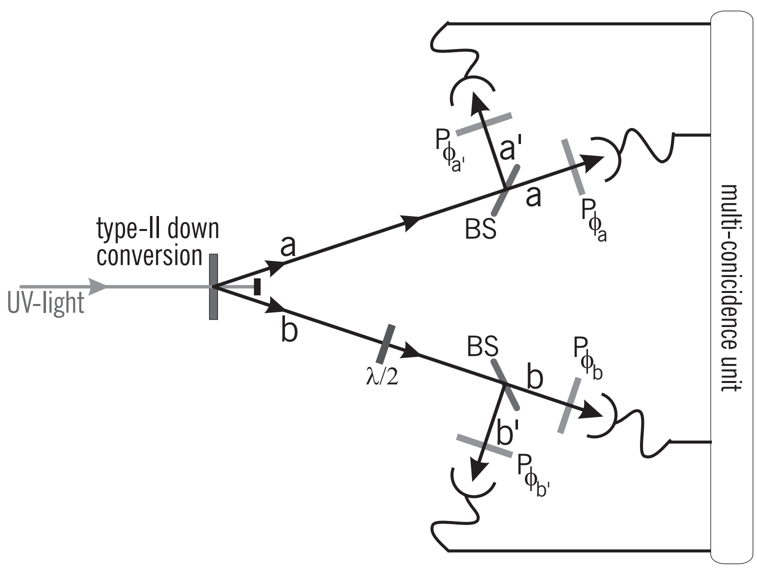

FIG. 1. Experimental set up to demonstrate the entanglement inherent in 4-photon emission from type-II paramteric down conversion. ($\lambda/2$-half wave plate to flip polarization, BS-beam splitter, and P_{ϕ_a} represents polarization analysis corresponding to the phase angle ϕ_a, etc.)